HUDSON PARK HIGH SCHOOL

GRADE 12

SUBJECT: MATHEMATICS PAPER 2

DATE: JUNE 2015 TOTAL: 150 MARKS

EXAMINER: MRS. C. SELKIRK TIME: 3 HOURS

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately.
3. A Diagram Sheet is provided. Please detach it and use it.
4. NB: Please STAPLE your submission in the following order:

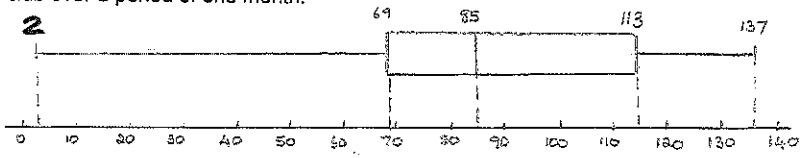
   Foolscap answer pages (on top)

   Diagram sheets (middle)

   Question paper (bottom)
5. Employ relevant formulae and show all working out.
6. (Non programmable and non graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. Start each new Question at the top of a new side of paper.

QUESTION 1 (15 marks)

1.1 The box and whisker plot below represents the distances run by athletes from a running club over a period of one month.



1.1.1 Comment on the distribution of distances for the club members if the mean is 95 km. (2)

1.1.2 Determine whether the minimum value for the club is an outlier. (3)

1.2 In the grid below, a, b, c, d, e, f *and* g represent values in a data set written in increasing order. No value in the data set is repeated.

a	b	c	d	e	f	g

Determine the values of a, b, c, d, e, f *and* g if:

- The maximum value is 42
- The range is 35
- The median is 23
- The difference between the median and upper quartile is 14
- The interquartile range is 22
- The difference between b and c is 6
- the mean is 25 (10) [15]

QUESTION 2 (10 marks)

A group of 50 Grade 12 learners were surveyed on the amount of money they spend on entertainment each month. The results are tabulated below:

Expenditure (rands)	Number of Matrics	Cumulative frequency
$0 \leq x \leq 50$	2	2
$50 < x \leq 100$	a	5
$100 < x \leq 150$	8	13
$150 < x \leq 200$	13	26
$200 < x \leq 250$	16	b
$250 < x < 300$	8	50

2.1 Determine the values of a *and* b. (2)

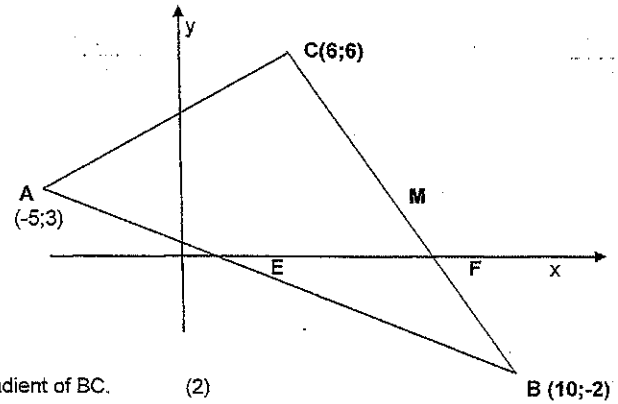
2.2 Use the graph paper provided on **Diagram Sheet A** to draw an ogive to represent this data. (4)

2.3 Use your graph to determine the median amount of money spent by these learners. Indicate your answer onto the graph. (2)

2.4 How many learners spent more than R 220 on entertainment in a month? (2) [10]

QUESTION 3 (17 marks)

A (-5 ; 3) ; C (6 ; 6) and B (10 ; -2) are the vertices of △ABC.



3.1 Calculate the gradient of BC. (2)

3.2 Determine the length of the line segment BC (leave your answer in simplest surd form, showing all working out). (4)

3.3 What are the coordinates at M, the midpoint of BC? (2)

3.4 D is a point such that line AD is parallel to BC. Determine the equation of line AD in the form

$ax + by + c = 0$ (3)

3.5 Determine the equation of a circle with centre M, which passes through points B and C. Give your answer in the form $(x-a)^2 + (y-b)^2 = r^2$ (3)

3.6 Does the point A lie inside, outside or on the circle in Question 3.5? Show all calculations to justify your answer. (3)

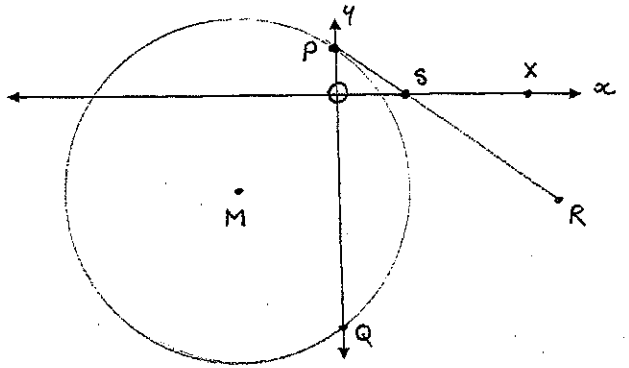
[17]

QUESTION 4 (20 marks)

The figure below shows a circle with equation

$x^2 + 6x + y^2 + 4y - 12 = 0$

The circle cuts the y axis at P and Q. PR is a tangent to the circle at P. M is the centre of the circle. S is the x-intercept for PR.



4.1 Determine:

4.1.1 the coordinates of M. (5)

4.1.2 the area of the circle (1)

4.1.3 the coordinates of P. (4)

4.1.4 the equation of the tangent at P. (3)

4.1.5 the size of $P\hat{S}X$. (2)

4.1.6 Hence or otherwise, find the size of $R\hat{P}O$. (2)

4.2 The circle is transformed as follows:
Reflected about the line y = 0 and the radius reduced by a factor of 2.
Give the equation of the transformed circle. (3)

[20]

QUESTION 5 (3 marks)

Use the identity cos (A-B) = cosA cosB + sinA sinB to show that:
sin (A + B) = sinA cosB + cosA sin B [3]

QUESTION 6 (16 marks)

Without using a calculator, showing ALL calculations:

6.1 If $x = 3\sin\theta$ and $y = 3\cos\theta$, find the value of $x^2 + y^2$ (3)

6.2 Simplify: $\sin(-x)\tan(-x + 180°)\cos(1260° + x) + \sin(90° + x)\cos(-x - 180°)$ (7)

6.3 Simplify: $\dfrac{\sin 150°.\cos 225°.\sin 260°}{\cos(-350°).\sin 315°}$ (6)

[16]

QUESTION 7 (19 marks)

7.1.1 Prove the identity: $\frac{2tanx-sin2x}{2sin^2x} = \tan x$ (5)

7.1.2 For which values of x will the above identity be undefined? (3)

7.2 If sin 61° = $\sqrt{p}$, express in terms of p:

7.2.1 cos 61° (2)

7.2.2 sin 241° (2)

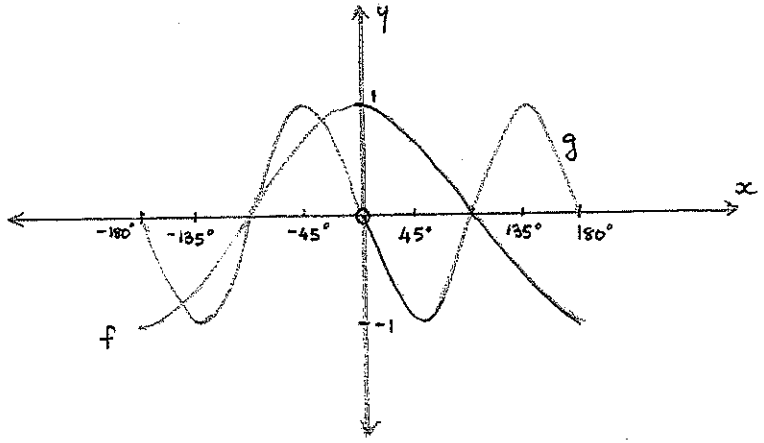
7.2.3 cos 122° (3)

7.2.4 cos 73°cos15° + sin73° sin 15° (4)

[19]

QUESTION 8 (12 marks)

In the diagram below the graphs of $f(x) = \cos x$ and $g(x) = -\sin 2x$ for $x \in$ [-180° ; 180°] are drawn.



8.1 Determine the x values where $f(x) = g(x)$. (6)

8.2 For which values of x will:

8.2.1 f(x) > g(x) for $x \in$ [0° ; 180°] (2)

8.2.2 $f(x).g(x) \leq 0$ (3)

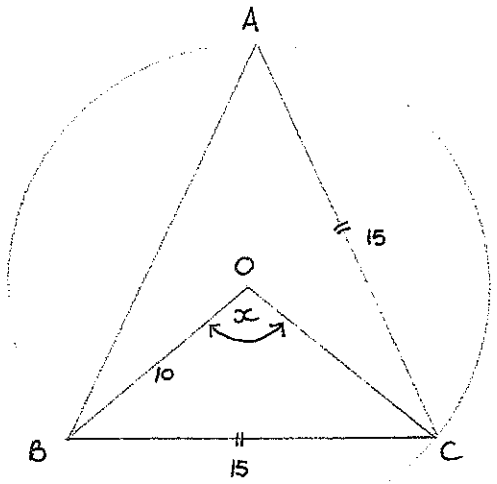
8.3 What is the equation of h(x) if h(x) = f(x) which has been shifted 20° to the left. (1)

[12]

THE DIAGRAMS FOR QUESTIONS 9 TO 11 ARE REPRODUCED ONTO DIAGRAM SHEETS B, C, D AND E

QUESTION 9 (6 marks)

In the diagram below, a circle with centre O passes through A, B and C. BC = AC = 15 units. BO and OC are joined. OB = 10 units and $\hat{BOC} = x$



Calculate:

9.1 The size of x (3)

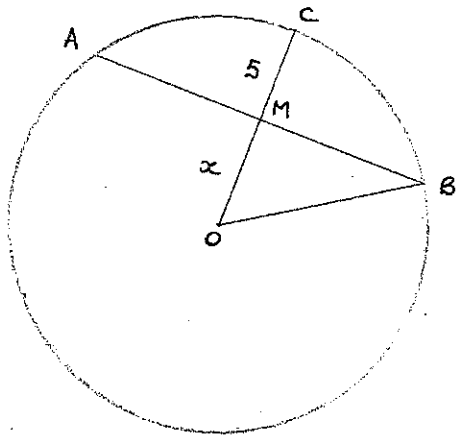
9.2 The size of $\hat{ACB}$ (with reasons) (3)

[6]

GIVE REASONS FOR YOUR ANSWERS IN QUESTIONS 10 AND 11.

QUESTION 10 (6 marks) TO BE COMPLETED ONTO DIAGRAM SHEET C

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. OM = x units, AB = 20 units. C lies on the circumference of the circle. MC is 5 units in length.



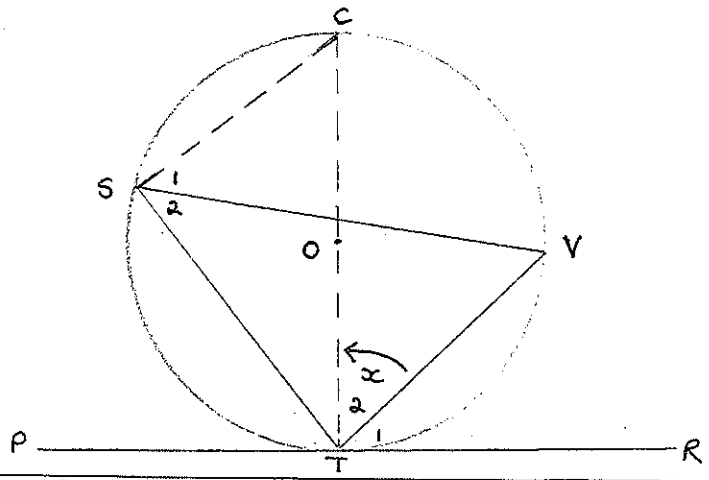
10.1 What is the length of MB? (1)
10.2 Give a reason why OM ⊥ AB. (1)
10.3 Write down the length of OC in terms of x (1)
10.4 Hence, calculate the value of x (3)
[6]

QUESTION 11 (15 marks) TO BE COMPLETED ONTO DIAGRAM SHEET D and E

11.1 Complete the statement of the following theorem:

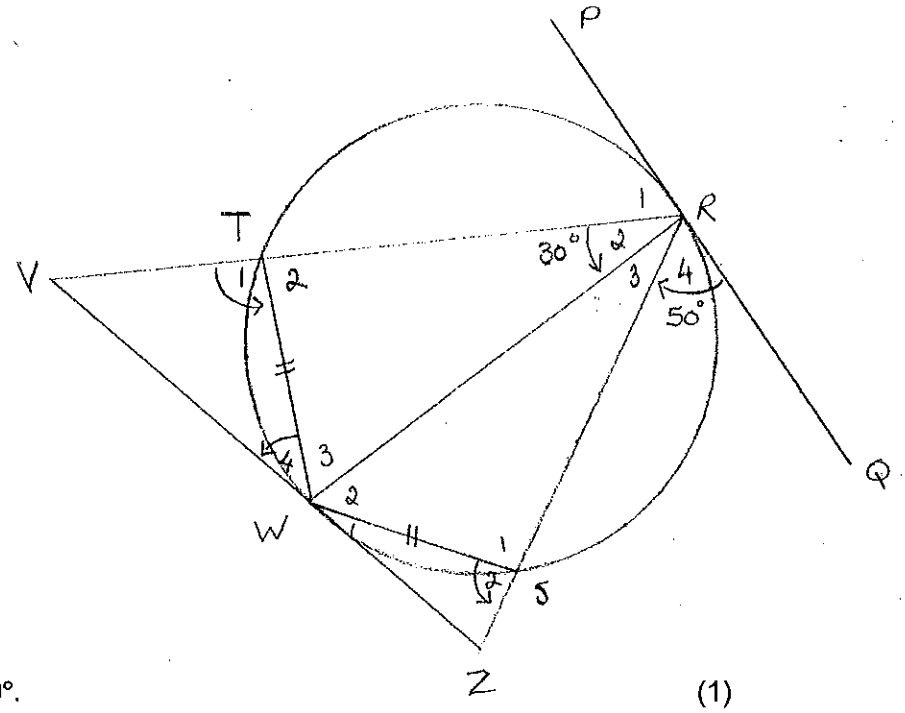
The exterior angle of a cyclic quadrilateral is equal to ..... (1)

11.2 In the diagram below, the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined. The partially completed proof of the theorem that states that $\hat{V}TR = V\hat{S}T$ is given below. Using the table below as a reference, **complete the proof of the theorem onto Diagram Sheet D.**
Construction: Draw diameter TC and join CS. Let $\hat{V}TC = \hat{T}_2 = x$ (5)



Statement	Reason
$\hat{S}_1 + \hat{S}_2 =$	
$\hat{T}_1 + \hat{T}_2 =$	
$\hat{T}_2 = \hat{S}_1 = x$	
$\therefore \hat{T}_1 = 90° - x$	
and $\hat{S}_2 = 90° - x$	
$\therefore V\hat{T}R = V\hat{S}T$	

11.3 In the figure, TSRW is a cyclic quadrilateral with TW = WS. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent t the circle at R. RW is joined. $\hat{R}_2 = 30°$ and $\hat{R}_4 = 50°$.



11.3.1 Give a reason why $\hat{R}_3 = 30°$. (1)

11.3.2 State with reasons, TWO other angles equal to 30° (3)

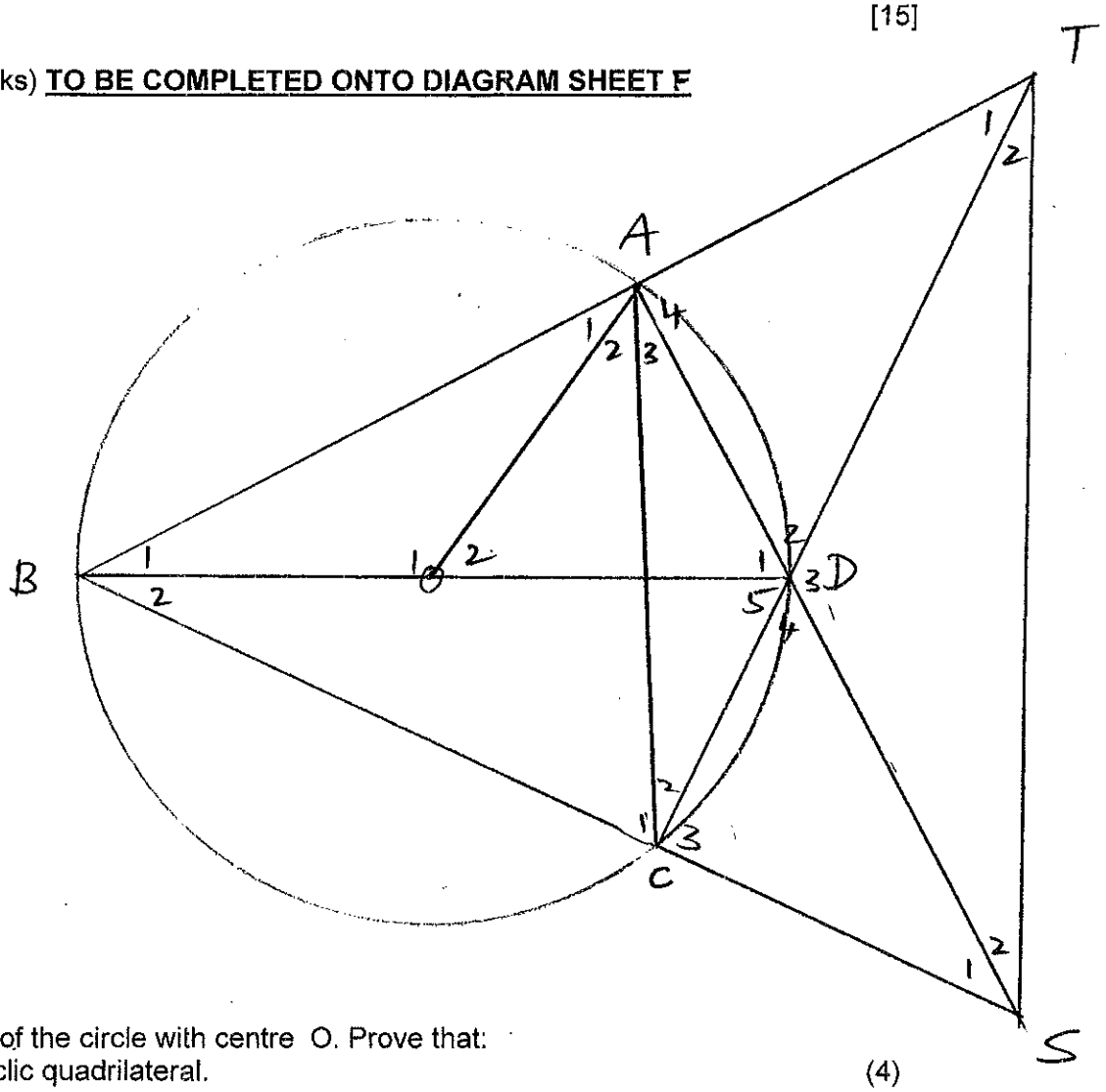
11.3.3 Determine, with reasons, the size of :

11.3.3.1 $\hat{S}_2$ (3)

11.3.3.2 $\hat{V}$ (2)

[15]

Question 12 (11 marks) **TO BE COMPLETED ONTO DIAGRAM SHEET F**



BOD is the diameter of the circle with centre O. Prove that:

12.1 ATSC is a cyclic quadrilateral. (4)

12.2 $A\hat{D}B = A\hat{T}S$ (3)

12.3 OA is a tangent to the circle through ATSC (4) [11]

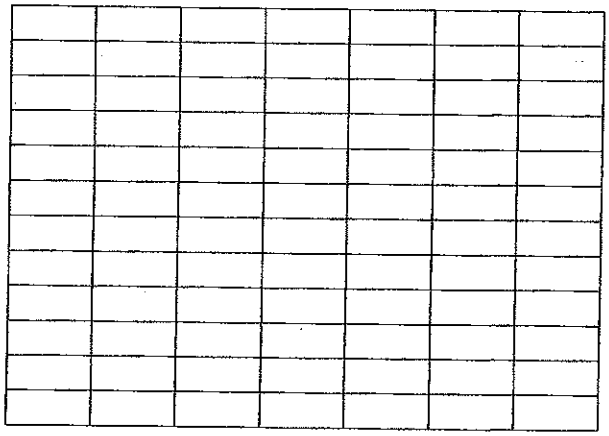
Diagram Sheet A

Name: _____

Question 2.1

Expenditure (rands)	Number of Matrics	Cumulative frequency
$0 \leq x \leq 50$	2	2
$50 < x \leq 100$	a	5
$100 < x \leq 150$	8	13
$150 < x \leq 200$	13	26
$200 < x \leq 250$	16	b
$250 < x < 300$	8	50

2.2



Expenditure (in rands)

2.3 _____

_____

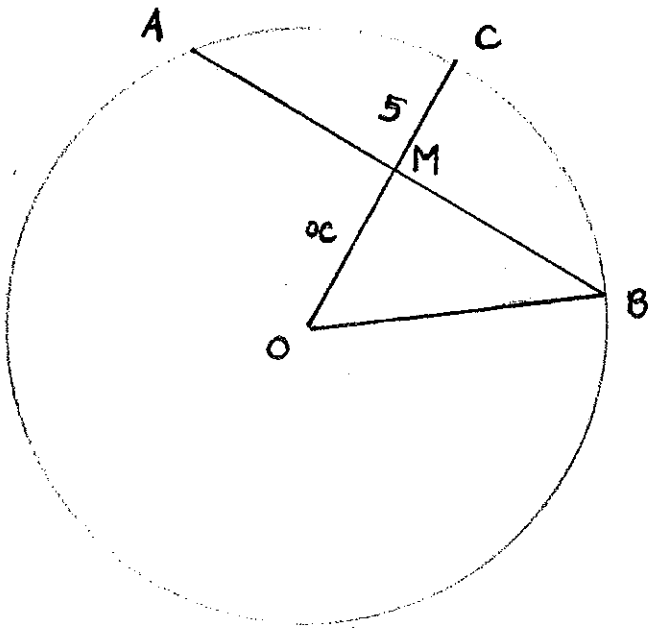
2.4 _____

_____

Diagram Sheet C

Name: _____

Question 10



10.1 _____ (1)

10.2 _____ (1)

10.3 _____ (1)

10.4 _____

_____

_____

_____

_____

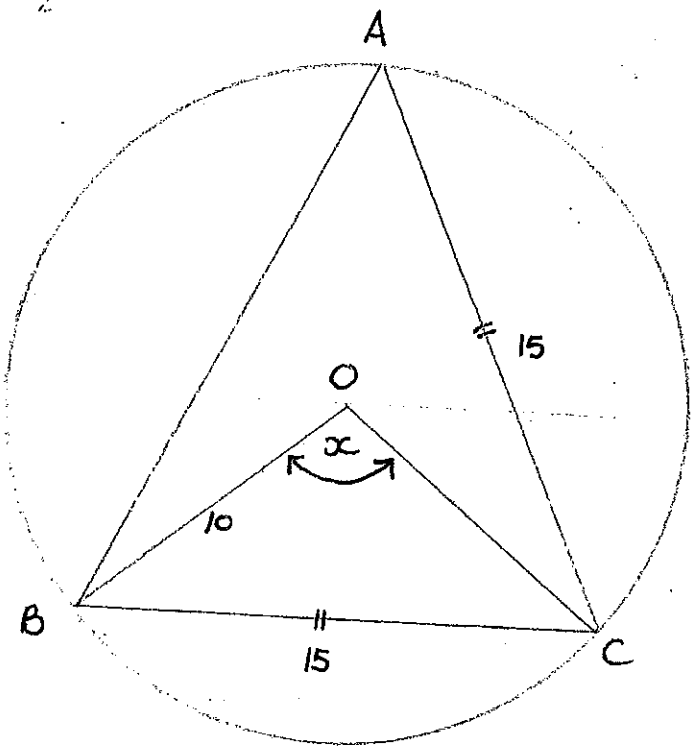
_____ (3)
[6]

Diagram Sheet B

Name: _____

Question 9

9.



9.1

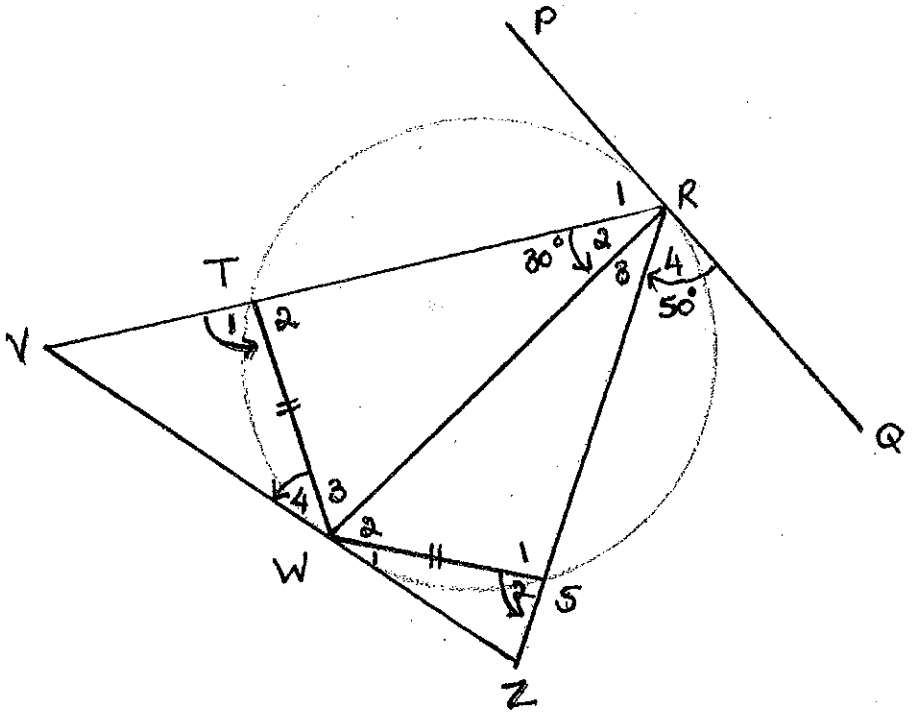
(3)

9.2

(3)

Diagram Sheet E

Name: _____



11.3.1 _____ (1)

11.3.2 _____

_____

_____

_____ (3)

11.3.3.1 _____

_____

_____ (3)

11.3.3.2 _____

_____ (2)

Diagram Sheet D

Name: _____

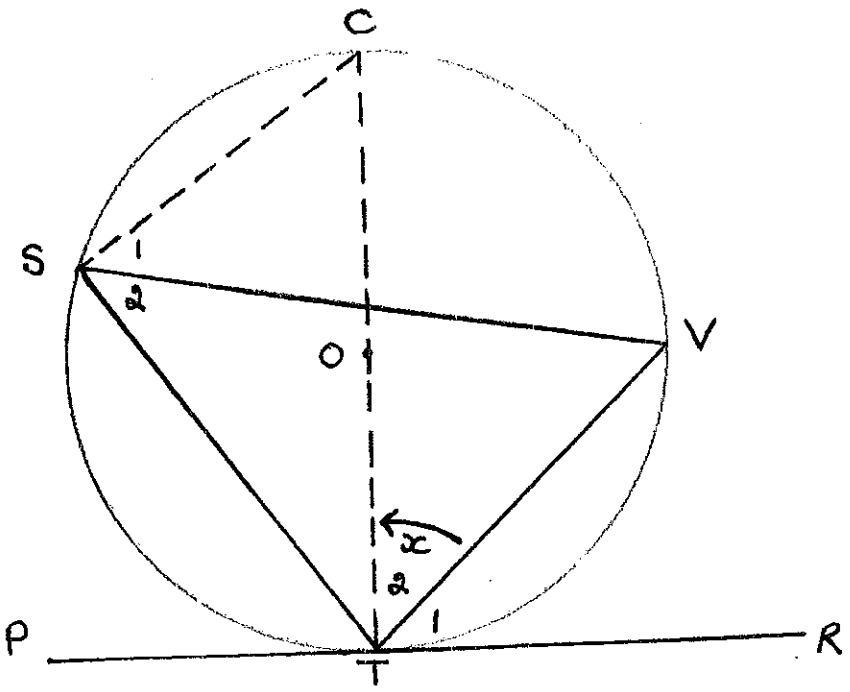
Question 11

11.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to .....

_____ (1)

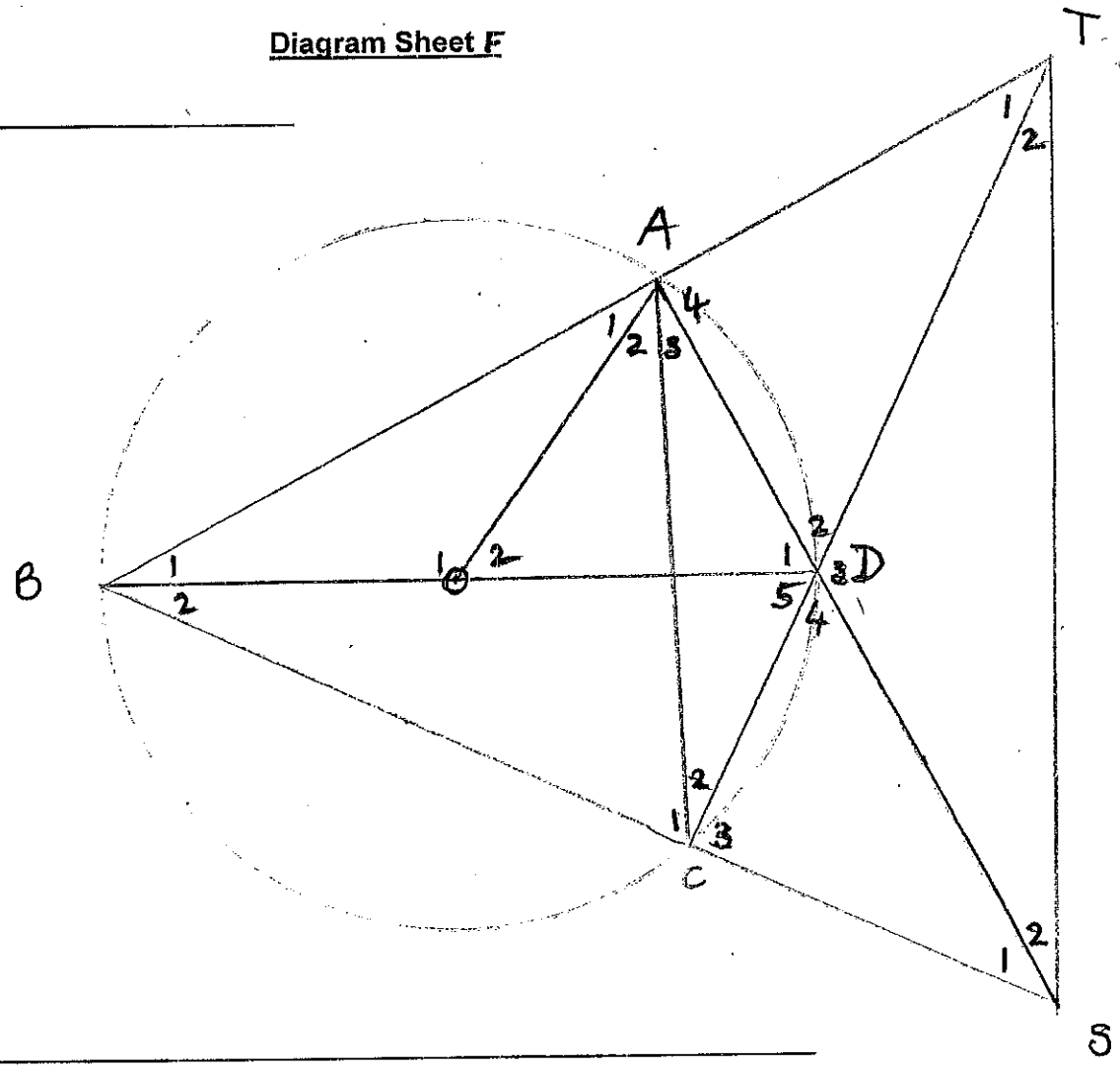
11.2 In the diagram below, the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined. **Construction: Draw diameter TC and join CV.** Let VTC = $T_2 = x$



Statement	Reason
$S_1 + S_2$ = ...	.....
$T_1 + T_2$ = ...	.....
$T_2 = S_1 = x$	.....
$\therefore T_1 = 90° - x$	
and $S_2 = 90° - x$	
$\therefore$ VTR = VST	

Diagram Sheet F

Name: _____



12.1 _____

_____

_____

_____

_____ (4)

12.2 _____

_____

_____ (3)

_____

12.3 _____

_____

_____

_____ (4)

[11]

5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r}\;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \; \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

